Project: Boolean Graphs

Name: ______________________________

Period: _________

1) Using graph paper and your calculator, draw a picture containing graphs of lines, parabolas, absolute values, square roots, and circles. The picture must contain between 5 to 10 different equations (10 is all your calculator will store). You must have one of each different function in your picture.
2) Record your equations in the table below.
3) Determine a domain to restrict each equation. This will stop the equation from continuing throughout your whole picture. Write your domain in the restriction column. Record your equation with this restriction in the denominator, just as it must appear in your calculator. Include all necessary parentheses.
4) Determine a good viewing window.

Title of graph: ______________________________ Viewing Window: ______________________

	Picture Part	Equation	Restriction	Enter into calculator (equation and domain)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Volume of Cone Problem **Name** ______________________________

In this activity we will explore the **volumes of cones** of different shapes. You will need to know the following geometry formulas to do this activity

Circumference of a circle = ________________________

Volume of a cone = ______________________________

<u>Part 1: The Physical Model</u>

The circle you were given has a radius of 10 cm. Its **circumference** is ________ cm. The length of string your group is using has a length of ________ cm. (All groups have different lengths of string.)

Directions:

1. Cut out the circle.
2. Place the string on the circumference of the circle and mark the circle at the ends of the piece of string.
3. Remove the piece of string and draw straight line segments connecting these marks with the center of the circle forming a sector .
4. Cut out this sector of the circle and put it away. (Don't throw it away!)
5. Form a cone by taping the remaining part of the circle with the cut edges touching, but not overlapping.
6. Draw your cone below.

7. The radius of the original circle was 10 cm. What part of the cone has a length of 10 cm? Label this part of your drawing above 10 cm.

In order to find the **volume of the cone** we need to find the ______________ and the ______________. Let's see if we can do this. Start by finding the **circumference of the circle** which is the **base of the cone**. (Don't measure it! Think about how it was formed.) The **circumference** is __________ cm. (Round all decimals to 2 places). Use this value to find the **radius of the base** (which is the radius of the cone). The **radius** is ________ cm. Put this value where it belongs in your drawing above. Now we need to find the **height of the cone**. Explain how we can do this.

____

Label the height of your cone **H.** Write and solve the equation you would use to find **H.**

The **height** of the cone is ____________ cm. Put this value where it belongs in your drawing above.

We now have enough information to find the **volume of the cone**. Find it! Put all of your values into the volume formula above and calculate it.

Volume = ______________________________ = ________________.

Summarize the information about your cone below.

Arc length (of cut out sector)	Radius of cone	Height of cone	Volume of cone
____________	____________	____________	____________

Share this information with the rest of the class by filling in the chart on the board.

Part 2: The Graphical Model

We want to explore the relationship between the **Arc Length** and the **Volume of the Cone**. Put the results of some of your classmates in the chart below. (The arc length values do not have to be in order.)

Arc Length											
Volume of Cone											

Put the **Arc Length** values into **L1** of your calculator and **Volume** into **L2**. Make a scatterplot of this relationship and sketch the graph below.

xmin: __________
xmax: __________
ymin: __________
ymax: __________

Part 3: Algebraic Model

We want to find the function which will go through the data points of the scatterplot. (This is the exciting part and when you are finished you will see the most complicated function you have ever seen!) Redraw your cone below and label the **slant height** 10 cm again.

Our **function** is going to define the **volume** as a function of **x**, where **x** is the **arc length** of the cut out sector. To do this we are going to follow the same steps that we did for your specific cone, only begin by letting the length of the string be **x**. Start by finding the **circumference of the circle** which is the **base of the cone**. (Think about how you found it above.) The **circumference** (in terms of **x**) is ____________________.
Use this expression to find an expression for the **radius of the base** (which is the radius of the cone).
The **radius** is __________________________. Put this expression where it belongs in your drawing above.
Now we need to find the **height of the cone**. Again, if the height of the cone is **H,** write an equation you would use to find **H** below, and solve this equation for **H**. (Do not try to simplify the expression!)

The **height** of the cone in terms of **x** is __________________________. Put this expression where it belongs in your drawing above.
We now have enough information to find the function for the **volume of the cone**. Put your values into the volume formula and write a function **V(x)** which represents the **volume of the cone.**

V(x) = ____

(Was I correct? Is this the "messiest" function you've ever seen?) Well it's time to see if your function is correct. <u>Carefully</u>, put your volume function into **Y1** of your calculator, cross your fingers, and graph it. If your function is correct, answer the questions below.

Part 4: Questions

1. What values of **x** make sense to the problem situation?

____

2. What arc length will give us the maximum volume? _____________________________________

3. What is the maximum volume? ________________________________

Note: You are going to be working with this function later today in your homework assignment. And we don't want to enter it again! So, let's save the equation, the window settings (and the mode) in a Graph Data Base. Then we can recall it when we need it.

If the circle goes through the three data points of the scatterplot, continue with the worksheet. If not, find your mistake! (Again, you may need to adjust your window to see all of the circle. Also, if the figure does not look "circular", try a **5:ZSquare** from the **Zoom** menu of your calculator).

Before proceeding, clear the Circle from your graph by selecting **2nd**, then **PRGM**, then **1:ClrDraw** (and **ENTER**).

4. Write the equation of the circle with the correct center and radius.

Equation: ____

III. Checking Your Solution:

1. Solve the equation of your circle for **y** and enter the <u>two</u> resulting equations into **Y1** and **Y2** of your graphing calculator.

2. Graph the equations and, if you have solved the Great Goody correctly, again you should see a circle going through the three points of your scatterplot!

3. If the circle does **<u>not</u>** go through your three points, find your algebra mistake and try it again!

4. Show your final graph to me to verify that the problem was solved correctly when you turn in your algebraic solution. (I will initial this sheet .) Attach this worksheet to the paper that contains all of your work.

Data Collection Activity - Heat It, then Cool It!

1. **The Problem**

A temperature probe is placed in a cup of hot water. It remains there for approximately 40 seconds, then it is removed from this cup and placed in a cup of cold water for another 40 seconds. We will find an algebraic function that models the temperature recorded by the probe over the entire 80 seconds, then answer some questions about the model.

2. **The General Solution (Newton's Law of Cooling/Heating)**

We want to find a function **T(t)** that models the temperature **T** of the probe at any time **t**, measured in seconds. Using a property of physics, called **Newton's Law of Cooling/Heating**, the temperature in an activity such as this can be modeled by an exponential function in the form:

$$T(t) = a \cdot b^{t} + c .$$

3. **The Specific Solution**

First, to find an algebraic function that models our temperature vs. time data, it is clear that we need to write a piecewise function, one rule for the first (approximately) 40 seconds, and another for the last 40 (approximately) seconds. Trace on the data to find the time that separates the two rules. (t= ______ secs)

To find both of these rules, we will use the property of Newton's Law of Cooling.

In order to find a model of the form $T(t) = a \cdot b^{t} + c$ for the first part of our data, we need to find the constants **a**, **b**, and **c**. We can find the constants **a** and **c** by "tracing" on our scatterplot.

First we will find the constant **c**. According to Newton's Law of Cooling/Heating, and the data collected, the value of **c** would be approximately ____________. (Hint: Think about the geometric transformations!)

To find **a**, record the temperature when t=0. (0 , _______)

Substitute this value, with the value of **c**, into our model, and solve for **a**. Show your work below.

a= ____________

To find **b**, the last constant in the model, we need another ordered pair. Trace on the data until you get to approximately 10 seconds. Record this ordered pair. (________ , _________)

Substitute these values into the equation (with the values of **a** and **c**, and solve for the last unknown constant **b**. Show your work below.

b= ____________

T(t)= ______________

To check your work, graph your equation with your scatterplot to see how it fits the first part of the data.

The equation that fits the second part of the data is similar to the first equation. However, since we are beginning with a time other than t=0, we need to apply a "horizontal shift". Therefore, the resulting form of this function is $T(t) = a \cdot b^{t-h} + c$, where h= ______. Use this form, and the hints given for the first part of the function, and find a rule that fits this part of the data in the scatterplot. Show all of your work on the next page. Again, graph this equation to check it.

a= ____________

b= ____________

c= ____________

T(t)= ________________

Finally, combine the results of the two parts of your rule, and write a **piecewise function** that models the temperature vs. time data.

$$T(t) = \begin{cases} \\ \\ \end{cases}$$

4. **Working with the Temperature Function T(t).**

a. Notice on your graph, that there are two times when the temperature is 130 degrees. Use your graph to determine these two times when the temperature is 130 degrees.

t = ____________ , t = ____________

b. These two t values can also be found algebraically. Using each part of your piecewise function, algebraically find the the times that the temperature is 130 degrees. Compare your answers with your answers determined graphically.

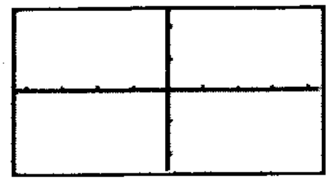
c. The second part of the piecewise function above, is in the form $T(t) = a \cdot b^{t-h} + c$. However, using properties of exponents, this function can be written in the form $T(t) = a \cdot b^{t} + c$. Apply the necesary properties of exponents, and rewrite the second part of your piecewise function, so the exponent on the value of b is just t, and not t-h. (Note: This will not change the values of b or c, but will change your value of a!) When done, graph this function to see if it also fits the data. Show the work below.

Inverse Trigonometric Functions

1. The Inverse Sine Function

Let $f(x) = \sin x$.

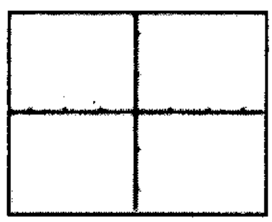
a. Graph f on the interval $[-2\pi, 2\pi]$.



b. Is $f(x) = \sin x$ a one-to-one function? ________ Why or why not? ______________________

Therefore, we must restrict the domain of f to make it one-to-one. We want to restrict the domain as close to the origin as possible. We restrict the domain of f to ___________ to make it a one-to-one function.

c. The inverse function of $f(x) = \sin x$ is $f^{-1}(x) = \sin^{-1} x$ (or sometimes written "arcsinx"). Sketch a graph of $f^{-1}(x) = \sin^{-1} x$ labelling the important values on the axes, and state the domain and range. Check your graph by graphing $y = \sin^{-1} x$ on your calculator.



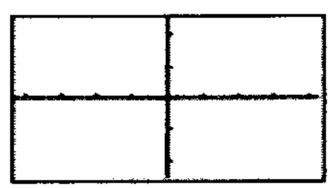
Domain: ________________

Range: ________________ (This is important!)

2. The Inverse Cosine Function

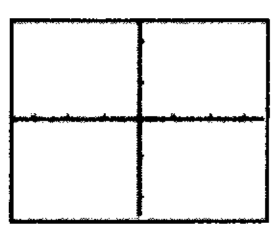
Let $f(x) = \cos x$.

a. Graph f on the interval $[-2\pi, 2\pi]$.



b. Since $f(x) = \cos x$ is not a one-to-one function we must restrict the domain of f to make it one-to-one. Therefore, we restrict the domain of f to ______________ to make it a one-to-one function.

c. The inverse function of $f(x) = \cos x$ is $f^{-1}(x) = \cos^{-1} x$ (or sometimes written "arccosx"). Sketch a graph of $f^{-1}(x) = \cos^{-1} x$ labelling the important values on the axes, and state the domain and range. Check your graph by graphing $y = \cos^{-1} x$ on your calculator.



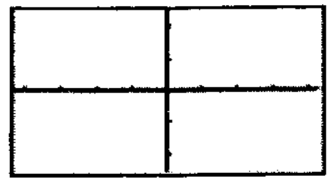
Domain: ________________

Range: ________________ (This is important!)

3. **The Inverse Tangent Function**

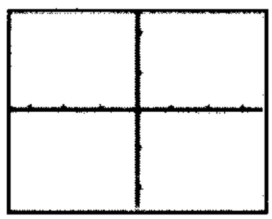
Let $f(x) = \tan x$.

a. Graph f on the interval $[-2\pi, 2\pi]$.



b. Since $f(x) = \tan x$ is not a one-to-one function we must restrict the domain of f to make it one-to-one. Therefore, we restrict the domain of f to ______________ to make it a one-to-one function.

c. The inverse function of $f(x) = \tan x$ is $f^{-1}(x) = \tan^{-1} x$ (or sometimes written "arctanx"). Sketch a graph of $f^{-1}(x) = \tan^{-1} x$ labeling the horizontal asymptotes, and state the domain and range. Check your graph by graphing $y = \tan^{-1} x$ on your calculator.



Domain: ________________

Range: ________________

4. Evaluate the following:

i. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ ii. $\cos^{-1}(-1)$ iii. $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

iv. $\tan^{-1}(-\sqrt{3})$ v. $\cos\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$ vi. $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$ (Be careful!)

5. Use a right triangle to evaluate:

i. $\sin(\tan^{-1}(3))$ ii. $\cos(\sin^{-1} x)$ iii. $\csc(\tan^{-1} x)$

Discovering Properties of Logarithms

There are certain properties of logarithms that are very important in applying mathematics to real world problems. All of these properties are based on the concept that "the logarithm of a number is an exponent". We already know two of these properties of logarithms.

1. $\log_b 1 =$ ________ 2. $\log_b b =$ ________

I. With the help of our graphing calculator, we will try to discover some other properties of logarithms. Written below are equations involving logarithms that **may** or **may not** be true. With your graphing calculator, put the expression on the left side of the equation into Y1 of your calculator, and the expression on the right side of the equation into Y2. From the graphs of each side of the equation decide whether you think the equation is **TRUE** (for all values of x) or **FALSE.** Set the WINDOW of your calculator so you have a viewing rectangle of:

Xmin:-3 Xmax:10 Ymin: -2 Ymax: 3

1. $\log(3+x) = \log 3 + \log x$ **TRUE** or **FALSE**
2. $\log(3 \cdot x) = \log 3 + \log x$ **TRUE** or **FALSE**
3. $\log(x^3) = (\log x)^3$ **TRUE** or **FALSE**
4. $\log(x^3) = 3(\log x)$ **TRUE** or **FALSE**
5. $(\log x)^3 = 3(\log x)$ **TRUE** or **FALSE**
6. $\log(x/4) = \log x / \log 4$ **TRUE** or **FALSE**
7. $\log(x-4) = \log x - \log 4$ **TRUE** or **FALSE**
8. $\log(x/4) = \log x - \log 4$ **TRUE** or **FALSE**

II. Look carefully at the statements above that you said were **TRUE**. Find patterns that allow you to write some **general rules** for logarithms.

1. $\log(a \cdot b) =$ ______________________________
2. $\log(a/b) =$ ______________________________
3. $\log(a^n) =$ ______________________________